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FOREIGN TECHNOLOGY DIVISION

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by

Stefan Staicu

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STUDIES

SUPERSONIC MOVEMENT AROUND A CROSS-SHAPED TAIL HAVING THE HORIZONTAL PLANE WITH SUPERSONIC LEADING EDGES, CONSIDERING THE FALLING OFF OF FLOW AT THE SUBSONIC LEADING EDGES OF THE PLATES.

by STEFAN STAICU
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In this work the supersonic flow around a thin cross-shaped tail is studied, with the incidente of the vertical plane antisimetrical, the horizontal plane having supersonic leading edges, taking into consideration the separation of the flow at the leading edges of the plates. As with the thin delta wing, with equal and opposed incidente on both its halves, in the case of the cross-shaped tail with three arms the flow separates at the edge of the plates in the form of horn shaped cones.
Treating the problem indirectly, as with the simple delta wing, here too we have succeeded in creating a theoretical study model, through which the distribution of pressure and the aerodynamic characteristics of the cross-shaped tail are arrived at.

1. INTRODUCTORY CONSIDERATIONS

In the following we will study the flow in a supersonic system around a thin cross-shaped tail, having the incidence of the vertical plane antisymmetrical, the horizontal plane having supersonic leading edges, taking into consideration the separation of flow at the leading edges of the plates. We will mark the incidence of the horizontal wing with α, the incidence of the higher part of the plate with β, and the incidence for the lower with $-\beta$.

We will consider the cross-shaped tail referred to a triangular axis system $Ox_1x_2x_3$ (fig. 1) with the origin in its peak, having the axis Ox_1, oriented behind the direction of the undisturbed flow U_∞.

As in the case of the ~~thin~~ simple delta wing (3), the flow is separated at the subsonic leading edges of the plates, giving birth to a system of vortexes which are located on the left and right of the plates, in the function of the signs of incidence α and β, producing an antisymmetrical movement. Thus, the flow is modified by the existence of two horn shaped vortexes of the same intensity and sign, situated antisimetrically to the face of the axis of symmetry. (fig, 1).

We will mark with $(-r'_0, t'_0)$, in the physical plane yOz, the coordinates of the nucleus of the vortex under which the horn is concentrated at the edge of the plate and we will take the effect of that vortex even on the surface of the tail.

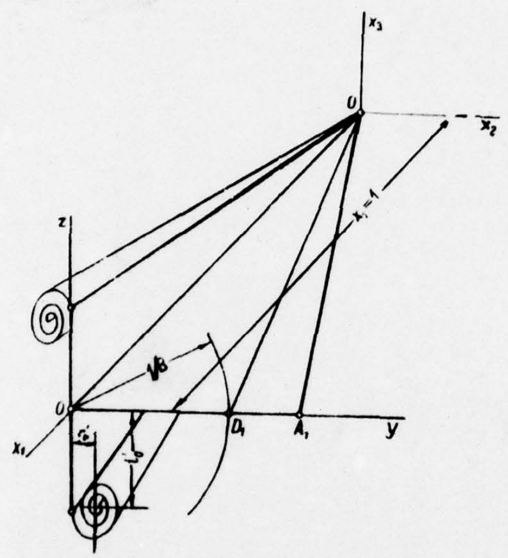

Fig. 1

That will manifest itself through the modification of the field by vertical and horizontal speeds on the plate or wing and through the avoidance of infinite speeds at the edge of the vertical plane predicted by classical liniar theory.

Making these considerations, the flow remains conical farther ~~father~~ away and can be treated using this method known from the theory of wings in supersonic system (2) in the hypothesis of negligent separation of flow at the edge of the plates. For this we will consider that the cross-shaped tail is

equivalent, from an aerodynamic point of view, with an imaginary thin one, with a suitable variation of incidence. To make the task easier, split the wing into three imaginary wing components:

a) the thin cross-shaped wing, having the incidence on the plate antisymmetrical and thus variable, such that it follows somewhat the phenomenon of changes of pressure and disturbance speeds on the plate near the leading edge. A thin imaginary cross-shaped tail with finite speed at the subsonic edge of the plate, and equal and of opposed signs on two sides, which does not agree with experimental indications, is obtained;

b) the "symmetrical thickness" of the cross-shaped wing, having the slopes of the plate equal and of the same sign with the incidence of the first wing components. This wing combined with a) will create another cross-shaped wing, which will have different pressures on the two halves of the plate, approaching somewhat the real situation;

c) the third wing will have "symmetrical thickness" with variable slope, however in such a manner that, combined with wing b), to obtain a nought mean slope, corresponds to a thin tail. This wing will have the role of compensating for the total aerodynamic effect from b) regarding the field of normal disturbance speeds.

Through the super[redacted]position of the three wings, we succeed in totaling up their aerodynamic effects, obtaining in that way a cross-shaped wing equivalent with the real one, in which the phenomenon of the falling off of the flow at the edges is considered.

2. DETERMINATION OF THE AXIS OF DISTURBANCE VELOCITIES

In the following we will determine the axis of disturbance velocities for the three imaginary wing components. For this we will first indicate the flow and plane variables used.

Starting from the physical plane yOz (fig.1) by coordinates

$$y = \frac{x_2}{x_1}, \qquad z = \frac{x_3}{x_1} \tag{1}$$

and making the transformation

$$\eta = \frac{y}{1 - B^2 z^2}, \quad \mathfrak{z} = \frac{z\sqrt{1 - B^2(y^2 + z^2)}}{1 - B^2 z^2} \qquad (B = \sqrt{M_\infty^2 - 1}), \tag{2}$$

the auxiliary plane is obtained

$$x = \eta + i\mathfrak{z}. \tag{3}$$

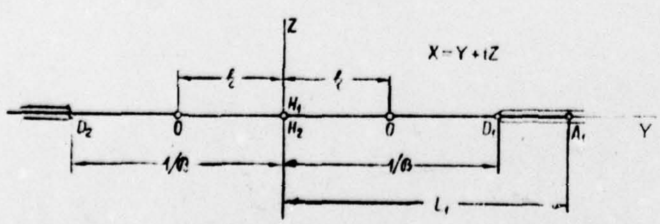

Fig. 2

Starting from these relations, we can deduce the height of the plate and ordinate of the position of the nucleus of the vortex created by the leading edge of the plate in plane x:

$$\mathfrak{h} = \frac{h}{\sqrt{1 - B^2h^2}}, \qquad \tau_0' = \frac{t_0'}{\sqrt{1 - B^2t_0'^2}}. \qquad (4)$$

We will pass from plane x into the complex plane X (fig. 2) through the intermediate conforming transformations

$$X^2 = x^2 + \mathfrak{h}^2, \qquad (5)$$

in which we will write the expressions of the axis of disturbance velocities.

2.1. Antisymmetrical thin cross-shaped tail with variable incidence on the surface of the plate. We will consider that, due to the existence of the vortexes, the vertical velocity on the wing with the supersonic edge is not modified, and the lateral one on the plate v' will have the variation

$$v_0 = -\beta_0 U_\infty \qquad (y = 0, \quad -t_0 \leqslant z \leqslant t_0), \qquad (6)$$

$$v' = v'(z) = -\beta'(z)\, U_\infty, \qquad [z = 0,\ y \in (-h, -t_0)\ U(h, t_0)], \qquad (6')$$

in such a manner that at the edge of plate we obtain the velocity

$$v_1 = -\beta_1 U_\infty. \qquad (7)$$

Due to the variation of lateral velocities on the plate, we will consider a distribution of elementary edges which will induce in the fixed point χ from the complex plane χ, obtained through the intermediate conforming transformations

$$\chi^2 = \frac{\mathcal{B}^2(\mathfrak{h}^2 + X^2)}{1 - \mathcal{B}X}, \tag{8}$$

the following expression of the axis of disturbance velocity:

$$d\mathcal{U}_i = q_i'(\chi_i') \ln \frac{\chi - \chi_i'}{\chi + \chi_i'} d\chi_i' - q_i''(\chi_i'') \ln \frac{\chi - \chi_i''}{\chi + \chi_i''} d\chi_i'', \tag{9}$$

in which

$$\chi_i' = \pm \sqrt{\frac{\mathcal{B}(\mathfrak{h} + T)}{1 - \mathcal{B}T}}, \quad \chi_i'' = \pm \sqrt{\frac{B(\mathfrak{h} - T)}{1 + \mathcal{B}T}}. \tag{10}$$

In order to obtain simpler results we will adopt a simpler distribution of sources in the work plane X:

$$q_i'(T) = \pm q_i \qquad (-T_0 \leqslant T \leqslant T_0). \tag{11}$$

Starting form this relation of divisions of intensities of sources, we will be able to deduce the axis velocity of the first wing components if we consider the contributions of the subsonic leading edges of the plate and of the supersonic horizontal plane:

$$\begin{aligned}\mathcal{U}_i = {} & A_{10} \frac{\sqrt{(X + \mathfrak{h})(1 - \mathcal{B}X)}}{X\sqrt{\mathcal{B}}} + \frac{2}{\pi} K_{10} \cos^{-1} \sqrt{\frac{(L + \mathfrak{h})(1 - \mathcal{B}X)}{(L - X)(1 + \mathcal{B}\mathfrak{h})}} + \\ & + \frac{2}{\pi} q_i \int_{T_0}^{0} \left(\cos h^{-1} \sqrt{\frac{(X + \mathfrak{h})(1 - \mathcal{B}T)}{(X - T)(1 + \mathcal{B}\mathfrak{h})}} - \right. \\ & \left. - \cos h^{-1} \sqrt{\frac{(X + \mathfrak{h})(1 + \mathcal{B}T)}{(X + T)(1 + \mathcal{B}\mathfrak{h})}} \right) dT,\end{aligned} \tag{12}$$

which, in the following calculations, becomes

$$\mathcal{U}_1 = A_{10}\frac{\sqrt{(X+\mathfrak{h})(1-\mathcal{B}X)}}{X\sqrt{\mathcal{B}}} + \frac{2}{\pi}K_{10}\cos^{-1}\sqrt{\frac{(L+\mathfrak{h})(1-\mathcal{B}X)}{(L-X)(1+\mathcal{B}\mathfrak{h})}} +$$

$$+\frac{2}{\pi}q_1\Big[(X-T_0)\cos h^{-1}\sqrt{\frac{(X+\mathfrak{h})(1-\mathcal{B}T_0)}{(X-T_0)(1+\mathcal{B}\mathfrak{h})}} +$$

$$+(X+T_0)\cos h^{-1}\sqrt{\frac{(X+\mathfrak{h})(1+\mathcal{B}T_0)}{(X+T_0)(1+\mathcal{B}\mathfrak{h})}} - 2X\cos h^{-1}\sqrt{\frac{X+\mathfrak{h}}{X(1+\mathcal{B}\mathfrak{h})}} -$$

$$-\frac{\sqrt{(X+\mathfrak{h})(1-\mathcal{B}X)}}{\sqrt{\mathcal{B}}}\Big(\cos^{-1}\sqrt{\frac{1-\mathcal{B}T_0}{1+\mathcal{B}\mathfrak{h}}} +$$

$$+\cos^{-1}\sqrt{\frac{1+\mathcal{B}T_0}{1+\mathcal{B}\mathfrak{h}}} - \cos^{-1}\frac{1-\mathcal{B}\mathfrak{h}}{1+\mathcal{B}\mathfrak{h}}\Big)\Big]. \qquad (13)$$

2.2. Cross-shaped tail having the plate of symmetrical thickness with equal slopes with the incidence of the first wing components. We will introduce the second cross-shaped tail, having the plate of symmetrical thickness in order to eliminate the accentuated peaks of pressure on its lower side. Proceeding in the same manner as with the simple delta wing (3), we will obtain in plane X the following expression of the axis of disturbance velocity:

$$\mathcal{U}_1 = \frac{2}{\pi}Q_{10}\cos h^{-1}\sqrt{\frac{X+\mathfrak{h}}{X(1+\mathcal{B}\mathfrak{h})}} +$$

$$+\frac{2}{\pi}q_1\int_{T_0}^{0}\Big(\cos h^{-1}\sqrt{\frac{(X+\mathfrak{h})(1-\mathcal{B}T)}{(X-T)(1+\mathcal{B}\mathfrak{h})}} + \cos h^{-1}\sqrt{\frac{(X+\mathfrak{h})(1+\mathcal{B}T)}{(X+T)(1+\mathcal{B}\mathfrak{h})}}\Big)\,dT, \qquad (14)$$

which becomes

$$\mathcal{U}_1 = \frac{2}{\pi}Q_{10}\cos h^{-1}\sqrt{\frac{X+\mathfrak{h}}{X(1+\mathcal{B}\mathfrak{h})}} +$$

$$+\frac{2}{\pi}q_1\Big[(X-T_0)\cos h^{-1}\sqrt{\frac{(X+\mathfrak{h})(1-\mathcal{B}T_0)}{(X-T_0)(1+\mathcal{B}\mathfrak{h})}} -$$

$$-(X+T_0)\cos h^{-1}\sqrt{\frac{(X+\mathfrak{h})(1+\mathcal{B}T_0)}{(X+T_0)(1+\mathcal{B}\mathfrak{h})}} -$$

$$-\frac{\sqrt{(X+\mathfrak{h})(1-\mathcal{B}X)}}{\sqrt{\mathcal{B}}}\Big(\cos^{-1}\sqrt{\frac{1-\mathcal{B}T_0}{1+\mathcal{B}\mathfrak{h}}} - \cos^{-1}\sqrt{\frac{1+\mathcal{B}T_0}{1+\mathcal{B}\mathfrak{h}}}\Big)\Big]. \qquad (15)$$

2.3. The cross-shaped tail of symmetrical thickness with the role of compensating the slope of the plate from 2.2. To this goal we will introduce a new distribution of source on the surface of the wing, which will reduce the wing at the average nought thickness corresponding to a thin wing.

The function chosen as the distribution of the intensities of the sources

$$q_i''(T) = k_i \frac{T}{\mathfrak{h}} \qquad (-\mathfrak{h} \leqslant T \leqslant \mathfrak{h}) \tag{16}$$

and we will obtain, similar to (15), the following expression of the axis of disturbance velocity:

$$\begin{aligned}
\mathcal{U}_i = & -\frac{2}{\pi} Q_{10} \cos h^{-1} \sqrt{\frac{X+\mathfrak{h}}{X(1+\mathcal{B}\mathfrak{h})}} + \\
& + \frac{2}{\pi} \frac{k_i}{\mathfrak{h}} \int_{\mathfrak{h}}^{0} T \left[\cos h^{-1} \sqrt{\frac{(X+\mathfrak{h})(1-\mathcal{B}T)}{(X-T)(1+\mathcal{B}\mathfrak{h})}} + \right. \\
& \left. + \cos h^{-1} \sqrt{\frac{(X+\mathfrak{h})(1+\mathcal{B}T)}{(X+T)(1+\mathcal{B}\mathfrak{h})}} \right] \mathrm{d}T = -\frac{2}{\pi} Q_{10} \cos h^{-1} \sqrt{\frac{X+\mathfrak{h}}{X(1+\mathcal{B}\mathfrak{h})}} + \\
& + \frac{k_i}{\pi \mathfrak{h}} \left\{ (X^2 - \mathfrak{h}^2) \cos h^{-1} \sqrt{\frac{(X+\mathfrak{h})(1-\mathcal{B}\mathfrak{h})}{(X-\mathfrak{h})(1+\mathcal{B}\mathfrak{h})}} - \right. \\
& - 2X^2 \cos h^{-1} \sqrt{\frac{X+\mathfrak{h}}{X(1+\mathcal{B}\mathfrak{h})}} + \\
& + \frac{\sqrt{(X+\mathfrak{h})(1-\mathcal{B}X)}}{2\mathcal{B}\sqrt{\mathcal{B}}} \left[\sqrt{2\mathcal{B}\mathfrak{h}} \left(\sqrt{1-\mathcal{B}\mathfrak{h}} - \sqrt{2} \right) + \right. \\
& \left. \left. + (1-\mathcal{B}\mathfrak{h}+2\mathcal{B}X) \left(\cos^{-1} \frac{1-\mathcal{B}\mathfrak{h}}{1+\mathcal{B}\mathfrak{h}} - \cos^{-1} \sqrt{\frac{1-\mathcal{B}\mathfrak{h}}{1+\mathcal{B}\mathfrak{h}}} \right) \right] \right\}.
\end{aligned} \tag{17}$$

Through the superposition of the three cross-shaped tails we realize a cross-shaped system with three arms, for which the

axis of disturbance velocity will have the expression

$$\mathcal{U} = \mathcal{U}_t + \mathcal{U}_t + \mathcal{U}_c. \tag{18}$$

3. The Determination of the Constants

Using the conditions at the limit for the disturbance velocities on the surface of the cross-shaped tail, we will determine the constants A_{10} and K_{10} which appear in the expression of the axis of disturbance velocity (13). Considering some sources concentrated in the place of the distribution, by intensity Q_t and position $Y = T_0'$,

$$Q_t = q_t T_0, \quad T_0' = \frac{1}{2} T_0, \tag{19}$$

we start from the compatibility relations of the disturbance velocities:

$$\mathrm{d}\mathcal{U} = -x\mathrm{d}\mathcal{V} = \frac{\mathrm{i}x}{\sqrt{1 - B^2x^2}}\mathrm{d}\mathcal{W}, \tag{20}$$

and we will write the equations of the conditions at the limit of the normal disturbance velocities in plane X:

$$\mathrm{Re}\, B\int_{\text{aripă}}^{\text{cercul Mach}} \sqrt{\frac{1/\mathcal{B}^2 - X^2}{\mathfrak{h}^2 - X^2}}\,\mathrm{d}\mathcal{U}_i' = \alpha U_\infty, \tag{21}$$

$$\mathrm{Re}\int_{\text{aripă}}^{\text{cercul Mach}} \frac{1}{\sqrt{X^2 - \mathfrak{h}^2}}\,\mathrm{d}\mathcal{U}_i' = v_0, \tag{21'}$$

in which U_1' is the axis of disturbance velocity of the first wing components in the case of the sources concentrated in $Y = T_0'$:

$$\mathcal{U}'_t = A_{10} \frac{\sqrt{(X+\mathfrak{h})(1-\mathcal{B}X)}}{X\sqrt{\mathcal{B}}} + \frac{2}{\pi} K_{10} \cos^{-1} \sqrt{\frac{(L+\mathfrak{h})(1-\mathcal{B}X)}{(1+\mathcal{B}\mathfrak{h})(L-X)}} +$$

$$+ \frac{2}{\pi} Q_t \left(\cos h^{-1} \sqrt{\frac{(X+\mathfrak{h})(1-\mathcal{B}T'_0)}{(X-T'_0)(1+\mathcal{B}\mathfrak{h})}} - \cos h^{-1} \sqrt{\frac{(X+\mathfrak{h})(1+\mathcal{B}T'_0)}{(X+T'_0)(1+\mathcal{B}\mathfrak{h})}} \right). \tag{22}$$

From (21) and (21') we deduce, accomplishing the calculations,

$$K_{10} = \frac{\alpha l U_\infty}{\sqrt{B^2 l^2 - 1}}, \tag{23}$$

$$\frac{Q_t}{\alpha U_\infty} = \sqrt{2\mathfrak{h}(1+\mathcal{B}\mathfrak{h})} \; \frac{\dfrac{v_0}{\alpha U_\infty} + \sqrt{\dfrac{1-\mathcal{B}^2\mathfrak{h}^2}{\mathcal{B}^2 L^2 - 1}} - \sqrt{\dfrac{(L-\mathfrak{h})(1-\mathcal{B}\mathfrak{h})}{2\mathfrak{h}(1+\mathcal{B}L)}}}{\sqrt{\dfrac{1+\mathcal{B}T'_0}{\mathfrak{h}-T'_0}} - \sqrt{\dfrac{1-\mathcal{B}T'_0}{\mathfrak{h}+T'_0}}}. \tag{23'}$$

The condition of finite velocity at the edges of the plate ($X = 0$) determines the constant A_{10} :

$$A_{10} = 0. \tag{24}$$

Furthermore, for the calculation of the constant q_t, we will start from the relation

$$q'_t(t) = \frac{t}{\sqrt{1 - B^2 t^2}} \frac{dv'}{dt}, \tag{25}$$

known in the theory of conical motion, and we will write the equations

$$v_1 - v_0 = - q_t \sqrt{1 - B^2 h^2} \int_{h_0}^{h} \frac{dt}{(1 - B^2 t^2)\sqrt{h^2 - t^2}}, \tag{26}$$

$$v_0 t_0 + v' t \Big|_{t_0}^{h} - \int_{t_0}^{h} t \, dv' = vh, \tag{26'}$$

from which we deduce the following relations:

$$v_0 - v_1 = q_t \cos^{-1}\frac{t_0}{h}\sqrt{\frac{1-B^2h^2}{1-B^2t_0^2}}, \tag{27}$$

$$Bh(v - v_1) = q_t \cos^{-1}\sqrt{\frac{1-B^2h^2}{1-B^2t_0^2}}. \tag{27'}$$

From (19), (27) and (27') we deduce the constant q_t:

$$\frac{q_t}{U_\infty} = \frac{\sqrt{2\mathfrak{h}(1+\mathcal{B}\mathfrak{h})}}{2T_0'}\left\{\alpha\left[\sqrt{\frac{1-\mathcal{B}^2\mathfrak{h}^2}{\mathcal{B}^2L^2-1}} - \sqrt{\frac{(L-\mathfrak{h})(1-\mathcal{B}\mathfrak{h})}{2\mathfrak{h}(1+\mathcal{B}L)}}\right] + \beta\right\}\left[\sqrt{\frac{1+\mathcal{B}T_0'}{\mathfrak{h}-T_0'}} - \sqrt{\frac{1-\mathcal{B}T_0'}{\mathfrak{h}+T_0'}} - \frac{\sqrt{2\mathfrak{h}(1+\mathcal{B}\mathfrak{h})}}{2T_0'}\left(\cos^{-1}\frac{t_0}{h}\sqrt{\frac{1-B^2h^2}{1-B^2t_0'^2}} - \frac{1}{Bh}\cos^{-1}\sqrt{\frac{1-B^2h^2}{1-B^2t_0'^2}}\right)\right]^{-1}. \tag{28}$$

We will calculate the constant k_t from the expression of axis velocity (13) starting from a similar relation with (26') for the lateral velocity of the plate of the third wing components:

$$v''\,t\Big|_0^h - \int_0^h t\,dv'' = -vh, \tag{29}$$

and we will deduce

$$5B^2h^2(v_1 - v) = k_t\sqrt{1-B^2h^2}\,[1-(1-B^2h^2)^{5/2}]. \tag{30}$$

That relation, together with (27), determines the constant k_t:

$$k_t = -\frac{5Bh\,q_t}{\sqrt{1-B^2h^2}\,[1-(1-B^2h^2)^{5/2}]}\cos^{-1}\sqrt{\frac{1-B^2h^2}{1-B^2t_0^2}}. \tag{31}$$

4. THE DISTRIBUTION OF PRESSURE AND AERODYNAMIC CHARACTERISTICS

The determination of the coefficient of pressure on the wing and on the plate is made using the formulas

$$C_{pa} = -2\frac{u_a}{U_\infty}; \qquad C_{pp} = -2\frac{u_p}{U_\infty}, \qquad (32)$$

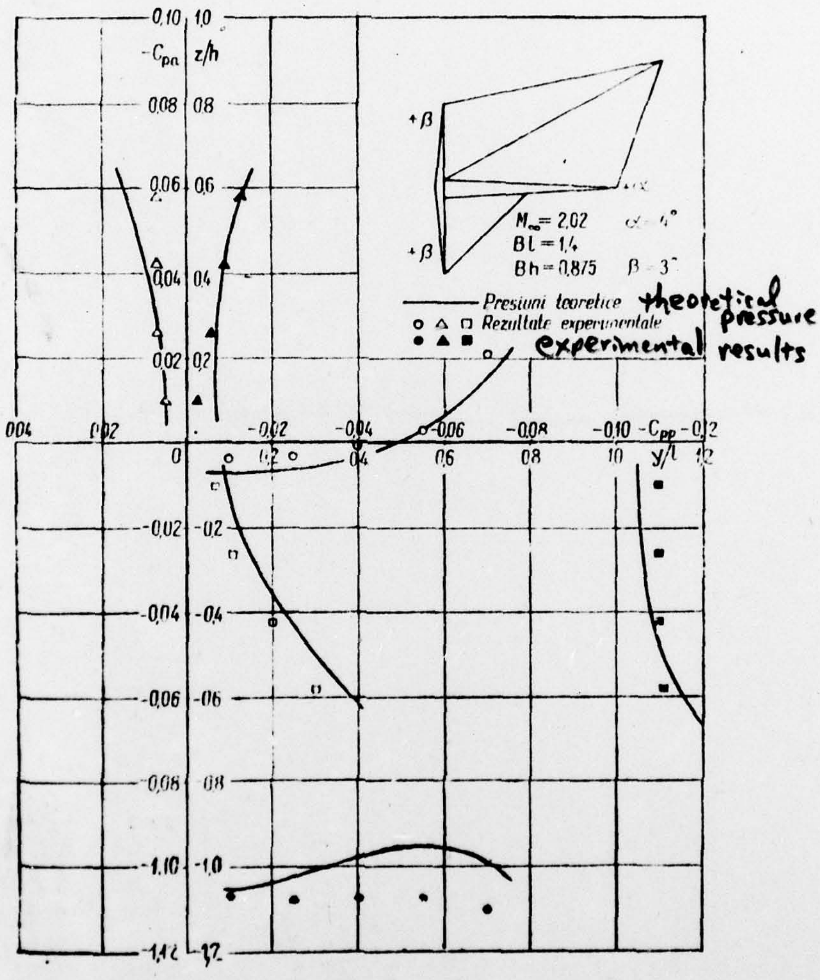

Fig. 3

in which the expressions of the axis of disturbance velocity given by (18) (fig. 3) ~~is~~ are introduced.

Coefficients of lift for the wing or half of the plate are obtained from the formulas

$$\frac{1}{2B} C_{zma} = \frac{2}{U_\infty} \int_{b}^{1/\mathcal{B}} u_{1a}(Y) \frac{Y\,dY}{\sqrt{Y^2 - b^2}} \tag{33}$$

for the region (m) contained in the interior of the circle Mach,

$$\frac{1}{2}\left(l - \frac{1}{B}\right) C_{zca} = \frac{1}{2}\left(l - \frac{1}{B}\right) \frac{4\alpha}{B} \tag{34}$$

for the outside,

$$C_{zp} = \frac{2(1 - \mathcal{B}^2 b^2)^{3/2}}{h U_\infty} \int_{-b}^{b} u_{1p}(Y) \frac{Y\,dY}{(1 - \mathcal{B}^2 Y^2)\sqrt{(1 - \mathcal{B}^2 Y^2)(b^2 - Y^2)}} \tag{35}$$

for the plate and

$$\frac{1}{2} l C_{za} = \frac{1}{2}\left(l - \frac{1}{B}\right) C_{zca} + \frac{1}{2B} C_{zma} \tag{36}$$

for the whole horizontal wing.

Coefficients of moment of roll will be given by the following formulas:

$$H C_{ma} = \frac{8}{3U_\infty \sqrt{L^2 - b^2}} \int_{b}^{L} u_{1a}(Y)\, Y\,dY \tag{37}$$

for the horizontal plane and

$$H C_{mp} = -\frac{4(1 - \mathcal{B}^2 b^2)}{3h U_\infty} \int_{-b}^{b} u_{1p}(Y) \frac{Y\,dY}{(1 - \mathcal{B}^2 Y^2)^2} \tag{38}$$

for the vertical plane, in which $u_{1a}(Y)$ and $u_{1p}(Y)$ are given by (18).

For the definition of the parameter $\frac{t_0}{h}$ we will notice first that the position of maximum distribution of pressure coincides with that of the center of the nucleus of the vortex, as is ascertained in another way from experience. However, making the calculation on the base of the distribution of chosen sources, it is ascertained that the peak of depressions on the higher side of the plate falls approximately to the right of the center of gravity of the intensities of the sources, having the position t_0 deduced from (19) in the form of

$$t_0' = \sqrt{\frac{3h^2(1 - B^2t_0^2) + t_0^2(1 - B^2h^2)}{3(1 - B^2t_0^2) + 1 - B^2h^2}}. \tag{39}$$

We will use, through the following, the formula

$$\frac{t_0'}{h} = \frac{1}{1 + 1{,}7(\beta \pm \Delta\beta)^{1/2}} \tag{40}$$

for the definition of the position of the center of the nucleus of the vortex, in which β is the incidence of the plate, and $\Delta\beta$ the supplementary incidence created by the interference between the wing and plate and which is proportional with α. Therefore, if $\alpha \to 0$, then $\Delta\beta \to 0$.

5. The case of concentrated sources

We will seek to solve the problem in a simplified way, presupposing that the vertical velocity on the plate presents

a sudden drop in the right of the nucleus of the vortex. This work is resolved from a hydrodynamic point of view through the introduction of concentrated sources in the points corresponding to the positions of the centers of the vortexes: t_0' and $-t_0'$, of intensities Q_t and $-Q_t$.

The expressions of the axis of disturbance velocities will be the following:

$$\mathcal{U}_i' = A_{10}\frac{\sqrt{(X+\mathfrak{h})(1-\mathcal{B}X)}}{X\sqrt{\mathcal{B}}} + \frac{2}{\pi}K_{10}\cos^{-1}\sqrt{\frac{(L+\mathfrak{h})(1-\mathcal{B}X)}{(1+\mathcal{B}\mathfrak{h})(L-X)}} + \\ + \frac{2}{\pi}Q_t\left[\cos h^{-1}\sqrt{\frac{(X+\mathfrak{h})(1-\mathcal{B}T_0')}{(X-T_0')(1+\mathcal{B}\mathfrak{h})}} - \cos h^{-1}\sqrt{\frac{(X+\mathfrak{h})(1+\mathcal{B}T_0')}{(X+T_0')(1+\mathcal{B}\mathfrak{h})}}\right] \quad (41)$$

for the wing lift power,

$$\mathcal{U}_t' = \frac{2}{\pi}Q_{10}\cos h^{-1}\sqrt{\frac{X+\mathfrak{h}}{X(1+\mathcal{B}\mathfrak{h})}} + \\ + \frac{2}{\pi}Q_t\left[\cos h^{-1}\sqrt{\frac{(X+\mathfrak{h})(1-\mathcal{B}T_0')}{(X-T_0')(1+\mathcal{B}\mathfrak{h})}} + \cos h^{-1}\sqrt{\frac{(X+\mathfrak{h})(1+\mathcal{B}T_0')}{(X+T_0')(1+\mathcal{B}\mathfrak{h})}}\right] \quad (42)$$

for the wing of symmetrical thickness from 2.2 and

$$\mathcal{U}_s' = \mathcal{U}_c \quad (43)$$

for the third wing component.

The distribution of pressure is obtained substituting in (32) the expression of U given in (18).

The aerodynamic coefficients are deduced in the same way as

in the case of sources of distribution, in which appear the constants A_{10}, K_{10}, Q_t, k_t deduced from the equations

$$A_{10} = 0, \tag{44}$$

$$v_0 t_0' + v_1(h - t_0') = vh, \tag{44'}$$

$$Q_t = \frac{(v_1 - v_0)\, t_0'}{\sqrt{1 - B^2 t_0'^2}} \tag{44''}$$

and from (23'), (30)

$$\frac{Q_t}{U_\infty} = \frac{\sqrt{2\mathfrak{h}(1+\mathcal{B}\mathfrak{h})}\left\{\alpha\left[\sqrt{\frac{1-\mathcal{B}^2\mathfrak{h}^2}{\mathcal{B}^2L^2-1}} - \sqrt{\frac{(L-\mathfrak{h})(1-\mathcal{B}\mathfrak{h})}{2\mathfrak{h}(1+\mathcal{B}L)}}\right]+\beta\right\}}{\sqrt{\frac{1+\mathcal{B}T_0'}{\mathfrak{h}-T_0'}} - \sqrt{\frac{1-\mathcal{B}T_0'}{\mathfrak{h}+T_0'}} - \frac{h-t_0'}{ht_0'}(1+\mathcal{B}\mathfrak{h})\sqrt{2\mathfrak{h}(1-\mathcal{B}\mathfrak{h})}}, \tag{45}$$

$$\frac{k_t}{Q_t} = \frac{5B^2h}{(1-B^2h^2)^{5/2}-1}\sqrt{\frac{1-B^2t_0'^2}{1-B^2h^2}}, \tag{45}$$

and the constant K_{10} is the same as that given **by** (23).

OBSERVATIONS

a) The **existence** and positions of the vortexes are in function β as well as α.

b) For the case of plates without incidence ($\beta = 0$), the wing however having the incidence α, likewise obtains an antisymmetrical flow with vortexes.

c) Making $A_{10} = 0$, we obtain from the expression of constant A_{10}, calculated from within the framework of liniar

theory (11):

$$A_{10} = \frac{2\mathfrak{h}\sqrt{2\mathcal{B}\mathfrak{h}}\,U_\infty}{\pi\sqrt{1+\mathcal{B}\mathfrak{h}}}\left\{\beta + \alpha\left[\sqrt{\frac{(L-\mathfrak{h})(1-\mathcal{B}\mathfrak{h})}{2\mathfrak{h}(1+\mathcal{B}L)}} - \sqrt{\frac{1-\mathcal{B}^2\mathfrak{h}^2}{\mathcal{B}^2L^2-1}}\right]\right\}, \quad (46)$$

with the condition like the cross-shaped tail with antisymmetrical plate to have finite velocities at the edges, avoiding the appearance of vortexes in the form of horns:

$$\frac{\beta}{\alpha} = \sqrt{\frac{1-\mathcal{B}^2\mathfrak{h}^2}{\mathcal{B}^2L^2-1}} - \sqrt{\frac{(L-\mathfrak{h})(1-\mathcal{B}\mathfrak{h})}{2\mathfrak{h}(1+\mathcal{B}L)}}. \quad (47)$$

In this relation we can deduce the supplementary incident $\Delta\beta$ induced by the wing on the plate whatever would be the real incident β of the plate of the tail:

$$\Delta\beta = \left[\sqrt{\frac{1-\mathcal{B}^2\mathfrak{h}^2}{\mathcal{B}^2L^2-1}} - \sqrt{\frac{(L-\mathfrak{h})(1-\mathcal{B}\mathfrak{h})}{2\mathfrak{h}(1+\mathcal{B}L)}}\right]\alpha, \quad (48)$$

which we will introduce in (40) for the definition of the nucleus of the vortex.

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